SANGOMA: WP3 Innovative DA techniques

Sanita Vetra-Carvalho   Peter Jan van Leeuwen

University of Reading

s.vetra-carvalho@reading.ac.uk

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Overview

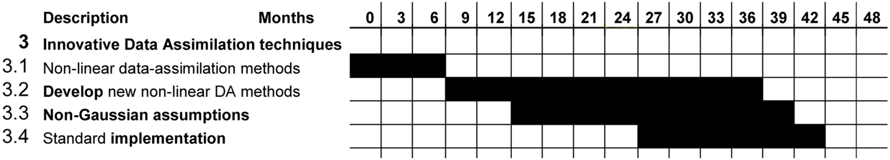
WP3 Participants

Work package number	3	**Start date or starting event:**			M0	
Work package title	Innovative Data Assimilation techniques					
Activity Type[1]	RTD					
Participant number	1	2	3	4	5	6
Participant short name	ULg	**UREAD**	AWI	TUD	CNRS	NERSC
Person-months per participant:	6	12	10	7	13	5

WP3 Objectives

- Collect information on current stochastic DA methods
- Develop a new DA method (non linear observation operator)
- Develop comparisons of non-Gaussian assumptions
- Develop algorithms for assesing observing systems (WP5)
- Produce living document of all the methods for users

Deliverables



D3.1: Report on current DA methods (M6)

D3.2: Living document on new DA methods (M6)

D3.3: Implementation of one common new DA method in all toolboxes (M36)

D3.4: Codes of new methods compliant with WP1 (M42)

D3.5: Final living document including uncertainty estimation (M48)

D3.1 currently includes

- Ensemble methods (various variants of EnSRF and pert. obs EnKF)
- Particle filters (PF) (equivalent weight and auxiliary)
- Gaussian mixture particle filter (GMPF)
- Various resampling methods for PFs and GMPF

Currently D3.2 includes proper orthogonal decomposition (POD) calibration method.

D3.3 Implementation of one common new DA method in all toolboxes

Current choice of non-Gaussian filters within Sangoma:

- Multivariate Rank Histogram filter (MRHF)
- Equivalent Weights Particle Filter (EWPF)
  - With linear observation operator $\mathbf{H}$
  - With non-linear observation operator $\mathcal{H}$

EWPF is a non-linear filter; however, it relies on linear(ized) $\mathbf{H}$ in the current form of the filter.

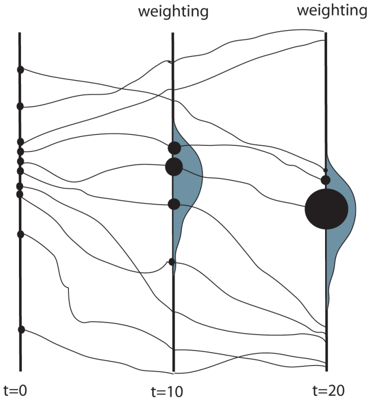
Properties of EWPF

EWPF is a particle filter which:

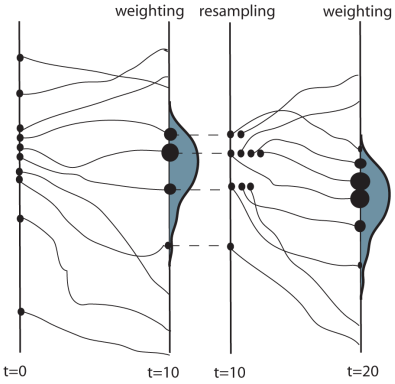
- Can represent the full posterior pdf.
- Uses future observations towards which the particles are nudged in between observation times.
- Is not degenerate due to equal weights of particles at analysis time.
- Uses a relatively small number of particles.

Schematics of particle filters

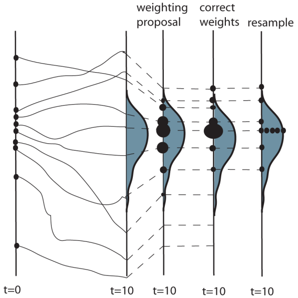
Importance Sampling



Sequential Importance Sampling



Schematics of EWPF



- In between observations
  - $\mathbf{x}_j^n = \mathcal{M}\left(\mathbf{x}_j^{n-1}\right) + nudging + d\beta_j^n$

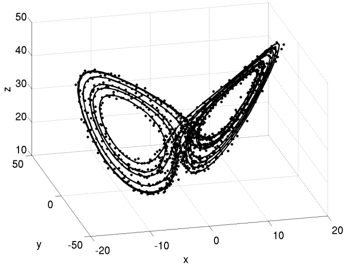
    $nudging = B(\tau)\left(\mathbf{y}^{(k)} - \mathcal{H}\left(\mathbf{x}_j^{(m-1)}\right)\right)$
  - $w_j^n = w_j^{n-1}\frac{p(\mathbf{x}^n|\mathbf{x}^{n-1})}{q(\mathbf{x}^n|\mathbf{x}^{n-1},\mathbf{y}^k)}$.
- At observation time
  - Max. weight each particle can achieve

    $C_j = w^{k-1}\max_{\mathbf{x}_j^k}\left[p(\mathbf{y}^k|\mathbf{x}_j^k)p(\mathbf{x}_j^k|\mathbf{x}_j^{k-1})\right]$
  - Keep particle if $C_j > C_{max}$
  - Find α such that $C_{max} - C_j = 0$
  - add stochastic noise

    $\mathbf{x}_j^n = \mathcal{M}\left(\mathbf{x}_j^{n-1}\right) + \alpha_j\mathbf{K}\left(\mathbf{y} - \mathcal{H}\left(\mathcal{M}\left(\mathbf{x}_j^{n-1}\right)\right)\right) -$
  - resample to have full set of particles.
  - $w_j = 1/N$.

Lorenz63 model

$$
\begin{aligned}
x^{n+1} &= x^n + \Delta t\sigma(y^n - x^n) + \xi_x^n \\
y^{n+1} &= y^n + \Delta t(\rho x^n - y^n - x^n z^n) + \xi_y^n \\
z^{n+1} &= z^n + \Delta t(x^n y^n - \beta^n z^n) + \xi_z^n
\end{aligned}
$$

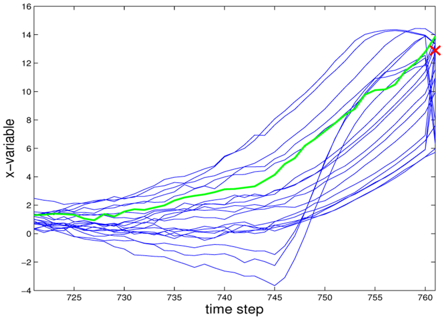


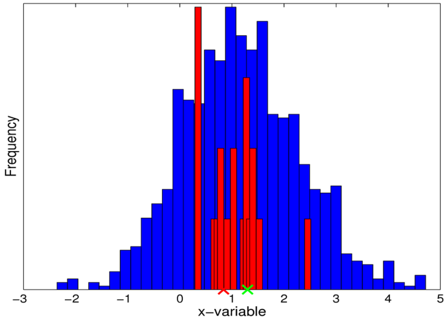
Settings:

- $dt = 0.01$
- $\mathbf{x}^0 = [1.508870, -1.531271, 25.4609]$
- $\sigma = 10,\ \rho = 28,\ \beta = 8/3$
- Stand. dev. of initial ensemble and observational error is $\sigma_R = \sqrt{2}$.
- Stand. dev. of model error is $\sigma_Q = \sqrt{2}\Delta t$.
- Numerical scheme: Euler

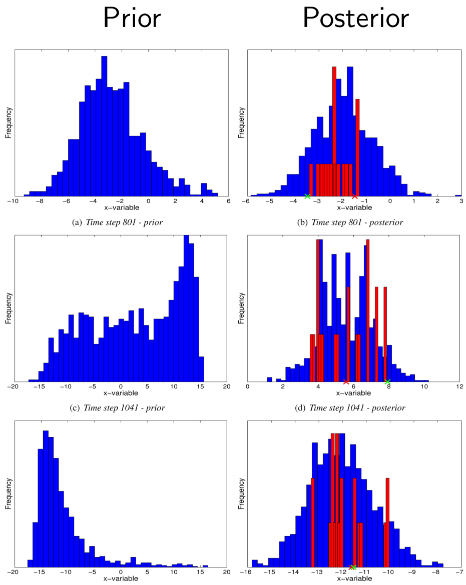
EWPF applied to Lorenz63...

Settings: $N = 20$, $\Delta t_{obs} = 40$





...EWPF applied to Lorenz63...



(a) *Time step 801 - prior* (b) *Time step 801 - posterior*

(c) *Time step 1041 - prior* (d) *Time step 1041 - posterior*

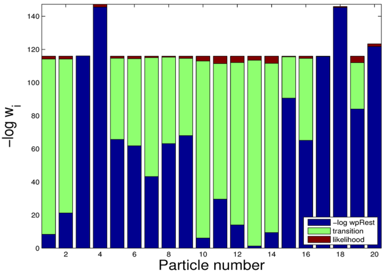
(e) *Time step 1481 - prior* (f) *Time step 1481 - posterior*

Equal particle weights (in log space) are given by

$$c_j = -w_j^{rest} + \left(\mathbf{x}_j^* - \mathcal{M}(\mathbf{x}_j^{n-1})\right)^T \mathbf{Q}^{-1} \left(\mathbf{x}_j^* - \mathcal{M}(\mathbf{x}_j^{n-1})\right) + \left(\mathbf{y}_j^k - \mathbf{H}(\mathbf{x}_j^*)\right)^T \mathbf{R}^{-1} \left(\mathbf{y}_j^k - \mathbf{H}(\mathbf{x}_j^*)\right).$$

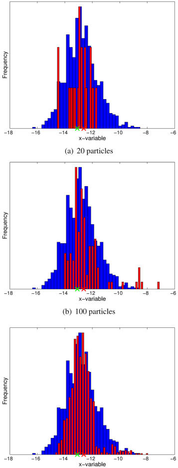
Particle weight contributions where

- Accumulated weights between observations are given by $w_j^{rest} = w_j^{k-1} \frac{p(\mathbf{x}^n|\mathbf{x}^{n-1})}{q(\mathbf{x}^n|\mathbf{x}^{n-1},\mathbf{y}^k)}$
- Transitional weights are given by $\left(\mathbf{x}_j^* - \mathcal{M}(\mathbf{x}_j^{n-1})\right)^T \mathbf{Q}^{-1} \left(\mathbf{x}_j^* - \mathcal{M}(\mathbf{x}_j^{n-1})\right)$ where $\mathbf{x}_j^n = \mathcal{M}\left(\mathbf{x}_j^{n-1}\right) + \alpha_j \mathbf{K}\left(\mathbf{y} - \mathcal{H}\left(\mathbf{x}_j^n\right)\right)$.
- Likelyhood weights are given by $\left(\mathbf{y}_j^k - \mathbf{H}(\mathbf{x}_j^*)\right)^T \mathbf{R}^{-1} \left(\mathbf{y}_j^k - \mathbf{H}(\mathbf{x}_j^*)\right)$



...EWPF applied to Lorenz63

Varying $N = 20, 100, 1000$

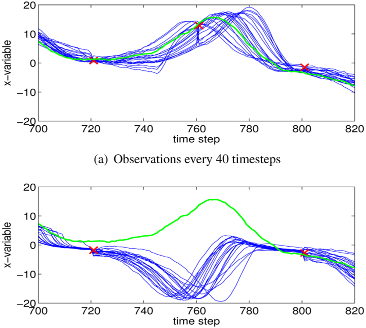


(a) 20 particles

(b) 100 particles

(c) 1000 particles

Varying $\Delta t^{obs} = 40, 80$



(a) Observations every 40 timesteps

(b) Observations every 80 timesteps

EWPF applied to BV model

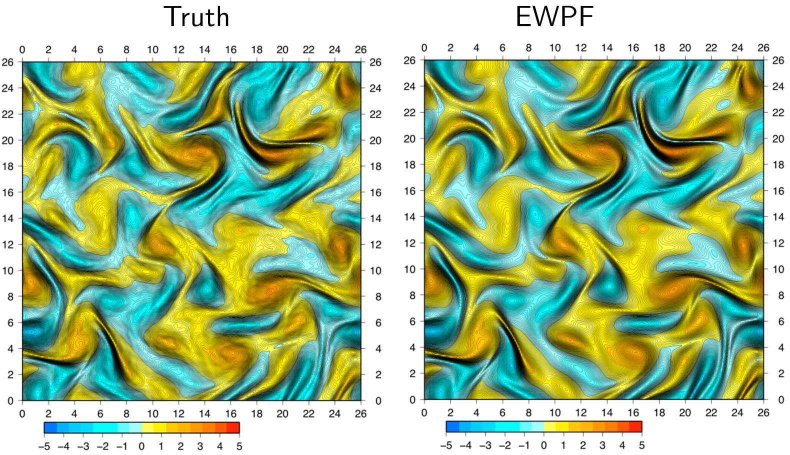
The highly nonlinear barotropic vorticity model is given by

$$\begin{aligned} \beta &= \frac{\partial q}{\partial t} - \frac{\partial \psi}{\partial y}\frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial q}{\partial y} \\ q &= \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 q}{\partial y^2} \end{aligned}$$

where q is the ccorticity field, ψ is the streamfunction, β is a model error term.

Grid: 256×256 gridpoints (state dim. ≈ 65000)
Scheme: Semi-Largrangian with $\Delta t = 0.04$ and $\Delta x = \Delta y = 1/256$.
Model error: Gaussian, zero mean, $\sigma = 0.01$, decoreltation lenghtscale 4 gridpoints.
Observations: Vorticity every 50 timesteps on every grid point with $\sigma = 0.05$.
EWPF: $N = 24$, nudging term $K = 0.1$.

...EWPF applied to BV model



EWPF with non-linear $\mathcal{H}$

Linearity of $\mathcal{H}$ is not important in between observations

$$\mathbf{x}_j^n = \mathcal{M}\left(\mathbf{x}_j^{n-1}\right) + B(\tau)\left(\mathbf{y}^{(k)} - \mathcal{H}\left(\mathbf{x}_j^{(m-1)}\right)\right) + d\beta_j^n.$$

The equivialent weights step at observation time relies on linear $\mathbf{H}$ to analytically find maximum weight possible of each particle and analytic α root to give equivalent weights for all particles that could reach the maximum weight.

$$C_j = -w_j^{rest} + \left(\mathbf{x}_j^* - \mathcal{M}(\mathbf{x}_j^{n-1})\right)^T \mathbf{Q}^{-1}\left(\mathbf{x}_j^* - \mathcal{M}(\mathbf{x}_j^{n-1})\right) + \left(\mathbf{y}_j^k - \mathbf{H}(\mathbf{x}_j^*)\right)^T \mathbf{R}^{-1}\left(\mathbf{y}_j^k - \mathbf{H}(\mathbf{x}_j^*)\right).$$

$$\mathbf{x}_j^* = \mathcal{M}\left(\mathbf{x}_j^{n-1}\right) + \alpha_j \mathbf{K}\left(\mathbf{y} - \mathcal{H}\left(\mathcal{M}\left(\mathbf{x}_j^{n-1}\right)\right)\right) + d\gamma_j^n$$

We have modified the equivalent weights step to work with a non-linear observation operator and currently testing it on Lorenz63 with $\mathcal{H}(\mathbf{x}) = xy$.

EMPIRE

Phil Browne and Simon Wilson have developed a simple way to incorporate any dynamical model within a data assimilation scheme using Message Passing Interface (MPI).

EMPIRE

- Uses simple code to pass state vectors from DA scheme to the dynamic model and vice versa
  - More complex if DA and/or model uses parallel computing.
- Requires user to define $\mathbf{H}, \mathbf{Q}, \mathbf{Qx}$ and $\mathbf{R}$ matrices.
- Is set up and can be used with various dynamical models:
  - Lorenz63, Barotropic Vorticity, Telemac, UM (operational global model), HadCM3.
- Is set up and can be used with following DA methods:
  - ETKF, EAKF, SIR and EWPF.

THANK YOU!

ANY QUESTIONS?